

Muon anomalous magnetic moment and positron excess at AMS-02 in a gauged horizontal symmetric model

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Abstract

We studied an extension of the standard model with a vector lepton doublet to explain the discrepancy in the muon $g - 2$ and explain the positron excess seen in the AMS-02 experiment. We introduced a gauge $SU(2)_H$ horizontal symmetry between the muon family and the 4th generation leptons. The neutral components of the 4th generation lepton doublet act as inelastic pseudo-Dirac dark matter. In this leptophilic model no antiproton excess predicted and the direct detection constraints are evaded. We get a boost factor of order 100 needed to explain the AMS-02 positron flux by dark matter annihilation into muons, by a Sommerfeld enhancement from Z exchange.

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1. INTRODUCTION

There exist a discrepancy at 3.6σ level between the experimental measurement [1] and standard model (SM) prediction [2–6] of muon anomalous magnetic moment.

$$\Delta a_\mu \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \times 10^{-10}, \quad (1)$$

where a_μ is the anomalous magnetic moment in the unit of $e/2m_\mu$. In the standard model, contribution of weak interaction to muon magnetic dipole moment (MDM) goes as $a_\mu^{\text{W}} \propto m_\mu^2/M_W^2$ and we have $a_\mu^{\text{SM}} = 19.48 \times 10^{-10}$ [7].

In minimal supersymmetric standard model (MSSM) [8, 9], we get contributions to muon MDM from neutralino-smuon and chargino-sneutrino loops. In all MSSM diagrams there still a m_μ suppression in $g - 2$ arising from the following cases: (a) In case of bino in the loop, the mixing between the left and right handed smuons is $\propto m_\mu \tan\beta$ (b) In case of wino-higgsino or bino-higgsino in the loop, the higgsino coupling with smuon is $\propto y_\mu$, so there is a m_μ suppression (c) In the case of chargino-sneutrino in the loop, the higgsino-muon coupling is $\propto y_\mu$, which again gives rise to m_μ suppression. Therefor in MSSM $a_\mu^{\text{MSSM}} \propto m_\mu^2/M_{\text{SUSY}}^2$, where M_{SUSY} is proportional to the mass of the SUSY particles in the loop.

One can evade the muon mass suppression in $g - 2$ with a horizontal gauge symmetry. In [10] a horizontal $U(1)_{L_\mu - L_\tau}$ symmetry was used in which muon $g - 2$ is proportional to m_τ and $a_\mu \propto m_\mu m_\tau / m_{Z'}^2$, where $L_\mu - L_\tau$ gauge boson mass $m_{Z'} \propto 100$ GeV gives the required a_μ . Models with $m_{Z'} \leq 1$ TeV are ruled out at LHC.

Recently there have been model independent analysis of the beyond SM particles which can give a contribution to a_μ [11]. The new physics needed to explain muon $g - 2$ has also been related to dark matter [12, 13] and the implication of this new physics in LHC searches has been studied [14].

In this paper we relate the anomaly of muon $g - 2$ to the excess of positron signal which extend upto 350 GeV observed by AMS-02 experiment [15]. An analysis of AMS-02 data suggest that a dark matter annihilation interpretation would imply that the annihilation final states are either μ or τ [17, 18]. Dark matter annihilation into e^\pm pairs would give a peak in positron signal, which is not seen in the positron spectrum. The AMS-02 experiment does not observe an excess, beyond the cosmic ray background, in the antiproton flux [24]. In this paper we introduced a 4th generation vector lepton doublet and a $SU(2)_H$ gauge symmetry between the muon family and 4th generation vector lepton doublet. In this

model Δa_μ generated by the charged 4th generation lepton and the $SU(2)_H$ gauge boson in the loop is,

$$\Delta a_\mu \propto \frac{m_\mu m_{L_4^-}}{M_{V_+}^2} \quad (2)$$

By taking the 4th generation lepton doublet mass, $m_{L_4^-} \sim 1$ TeV, and the $SU(2)_H$ gauge boson mass $M_{V_+} \sim 2$ TeV, we get the required contribution to Δa_μ (Eq.1).

In this model the neutral components of L_4 called $L_{4L,4R}^0$ act as dark matter (DM). The annihilation through the $SU(2)_H$ gauge boson V_3 produces μ final state, which is used to explain AMS-02 [15] positron data by taking the dark matter mass $m_{\chi_1} \sim 1$ TeV. Assuming the V_3 mass to be $M_{V_3} \approx 2m_{\chi_1}$, we get a resonant enhancement [21–23] in the annihilation cross-section (CS) that is used to get correct relic density $\Omega h^2 = 0.1199 \pm 0.0027$ [25, 26]. To satisfy the AMS-02 data [15], we require a boost factor of $O \sim (100)$ [18–20], that comes from Sommerfeld enhancement [27] in our model.

This paper is organized as follows: In Sec.2, we describe the model and dark matter and neutrino mass arising. In Sec.3 we discuss the fit to AMS-02 data. In Sec.4, we study the one-loop diagram which contribute to muon $g - 2$ in the model. We give our conclusion in Sec.5.

2. MODEL

In addition to the weak $SU(2)_W$ doublets $L_L = (\nu_i, E_i)_L^T$, $Q_L = (U_j, D_j)_L^T$, and $SU(2)_W$ singlets E_{iR} , U_{jR} , D_{jR} , where $(i = e, \mu, \tau)$ and $(j = u, d, c, s, t, b)$, we introduce two extra $SU(2)_W$ doublets,

$$L_{4L} = \begin{pmatrix} L_4^0 \\ L_4^- \end{pmatrix}_L, \quad L_{4R} = \begin{pmatrix} L_4^0 \\ L_4^- \end{pmatrix}_R, \quad (3)$$

with $Y = -1$, which have a vector coupling with the SM gauge bosons W_μ

$$ig \bar{L}_{4L} \gamma^\mu \frac{\tau}{2} \cdot W_\mu L_{4L} + ig \bar{L}_{4R} \gamma^\mu \frac{\tau}{2} \cdot W_\mu L_{4R} \quad (4)$$

and similarly for Z_μ boson (here g denotes the $SU(2)_W$ weak coupling). The vector coupling of the 4th generation $SU(2)_W$ lepton doublets ensures anomaly cancellation without introducing 4th generation of quarks doublets. In addition to the vector-like lepton generation, we also add three right handed neutrinos (ν_{iR}) in the model.

We introduce a $SU(2)_H$ horizontal gauge symmetry between (L_{4L}, L_{4R}) and the muon family $L_\mu = (\nu_\mu, \mu^-)_L$, μ_R and $\nu_{\mu R}$, which can be organized in the form of $SU(2)_H$ doublets as follows,

$$L_H^1 = \begin{pmatrix} \nu_{\mu L} \\ L_{4L}^0 \end{pmatrix}, L_H^2 = \begin{pmatrix} \nu_{\mu R} \\ L_{4R}^0 \end{pmatrix}, L_H^3 = \begin{pmatrix} \mu_L^- \\ L_{4L}^- \end{pmatrix}, L_H^4 = \begin{pmatrix} \mu_R^- \\ L_{4R}^- \end{pmatrix}, \quad (5)$$

These doublets couple to the $SU(2)_H$ horizontal symmetry gauge bosons $V_\mu^\pm = (V_\mu^1 \mp iV_\mu^2)/\sqrt{2}$ and $V_{3\mu}$ as,

$$\mathcal{L}_V = \frac{1}{2} i g_H \bar{L}_H^\alpha \tau \cdot V_\mu L_H^\alpha \quad (\alpha = 1, 2, 3, 4) \quad (6)$$

where τ is the 2×2 matrix representation of $SU(2)_H$, α is the $SU(2)_H$ family index (Eq.5), and g_H represents the $SU(2)_H$ gauge coupling. Here V_μ^\pm is the $SU(2)_H$ charge changing boson, which connect the L_μ and the L_4 families and $V_{3\mu}$ is the “neutral” current gauge boson of $SU(2)_H$. To give masses to the $SU(2)_H$ gauge bosons triplet V_μ^a , we introduce a $SU(2)_H$ scalar doublet ξ , which has no $SU(2)_H$ invariant Yukawa couplings with the SM and the 4th generation leptons.

In our model we also add a scalar triplet Δ with $Y = 2$, which generate the Majorana mass for the neutrinos and 4th generation neutral leptons, that are the dark matter in this model. The scalar fields in the model including the SM higgs doublet ϕ can be parametrized as,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \end{pmatrix}, \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}, \quad (7)$$

with $\phi^0 = (\phi + v_\phi + i\zeta)/\sqrt{2}$, $\xi^0 = (\xi + v_\xi + i\omega)/\sqrt{2}$, and $\Delta^0 = (\Delta + v_\Delta + i\varsigma)/\sqrt{2}$. Here v_ϕ , and v_Δ are the vacuum expectations values (vevs) of $SU(2)_W$ doublet and triplet higgs respectively. The vev of $SU(2)_H$ doublet higgs is denoted by v_ξ . We also introduce a $SU(2)_W$, $SU(2)_H$ singlet scalar η , which gives a Dirac mass to the 4th generation leptons L_4 . The quantum numbers of the extra particles under the SM groups $SU(3)$, $SU(2)_W$ and, $U(1)_Y$ are given in Table (I), and $SU(2)_H$ quantum numbers are given in Table (II).

	L _{4L}	L _{4R}	η	Δ
SU(3)	1	1	1	1
SU(2) _W	2	2	1	3
U(1) _Y	-1	-1	0	2

TABLE I: SM quantum numbers of the vector like particles and singlet higgs scalar.

	L _H ¹	L _H ²	L _H ³	L _H ⁴	ξ
SU(2) _H	2	2	2	2	2

TABLE II: Quantum numbers of the vector like particles and higgs scalar under SU(2)_H.

The gauge couplings of the scalar fields ϕ, ξ, Δ and η are given by the Lagrangian:

$$\begin{aligned} \mathcal{L}_S = & |(\partial_\mu \phi + i\frac{g}{2}\tau \cdot W_\mu \phi + ig'B_\mu \phi)|^2 + \text{Tr}|(\partial_\mu \Delta + i\frac{g}{2}[\tau \cdot W_\mu, \Delta] + ig'B_\mu \Delta)|^2 \\ & + |\partial_\mu \eta|^2 + |(\partial_\mu \xi + i\frac{g_H}{2}\tau \cdot V_\mu \xi)|^2. \end{aligned} \quad (8)$$

The masses of the gauge bosons generated after the corresponding scalars take their vevs are,

$$\begin{aligned} M_W^2 = \frac{g^2}{4}(v_\phi^2 + 2v_\Delta^2), \quad M_Z^2 = \frac{g^2 \text{Sec}^2 \theta_W}{4}(v_\phi^2 + 4v_\Delta^2), \quad M_A^2 = 0, \\ M_{V^+}^2 = \frac{g_H^2}{4}v_\xi^2, \quad M_{V_3}^2 = \frac{g_H^2}{4}v_\xi^2. \end{aligned} \quad (9)$$

and so the electroweak ρ parameter gets modified as,

$$\rho = \frac{1 + 2v_\Delta^2/v_\phi^2}{1 + 4v_\Delta^2/v_\phi^2} \quad (10)$$

we take the vev of the triplet higgs Δ , $v_\Delta \sim O(1)\text{GeV}$ due to the requirement that the electroweak ρ parameter, $\rho = 1.0004_{-0.0011}^{+0.0029}$ [7].

The renormalizable potential of the scalars field named $V(\phi, \xi, \Delta, \eta)$ which is invariant under SU(2)_W \times U(1)_Y \times SU(2)_H, can be written as,

$$\begin{aligned} V = & -\mu_\phi^2(\phi^\dagger \phi) + \frac{\lambda_\phi}{2}(\phi^\dagger \phi)^2 - \mu_\xi^2(\xi^\dagger \xi) + \frac{\lambda_\xi}{2}(\xi^\dagger \xi)^2 + \mu_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \frac{\lambda}{2}[\text{Tr}(\Delta^\dagger \Delta)]^2 + \frac{\lambda_\Delta}{2}[\text{Tr}(\Delta^\dagger \Delta)]^2 \\ & - \mu_\eta^2 \eta^2 + \frac{\lambda_\eta}{2} \eta^4 + \frac{\lambda_{\phi\xi}}{2}(\xi^\dagger \xi)(\phi^\dagger \phi) + \lambda_{\xi\Delta_1}(\xi^\dagger \xi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_{\xi\Delta_2} \xi^\dagger \Delta \Delta^\dagger \xi + \lambda_{\Delta\eta} \eta^2 \text{Tr}(\Delta^\dagger \Delta) \\ & + \frac{\lambda_{\eta\phi}}{2} \eta^2(\phi^\dagger \phi) + \frac{\lambda_{\eta\xi}}{2} \eta^2(\xi^\dagger \xi) + \lambda_{\phi\Delta_1}(\phi^\dagger \phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_{\phi\Delta_2} \phi^\dagger \Delta \Delta^\dagger \phi + \frac{1}{\sqrt{2}}[\mu_{\phi\Delta} \phi^T i\tau_2 \Delta^\dagger \phi + \text{h.c.}] \end{aligned} \quad (11)$$

In our model, the parameters in the potential are chosen such that the vevs of the scalars are,

$$v_\phi^2 + 2v_\Delta^2 = (174 \text{ GeV})^2,$$

$$v_\xi = 32 \text{ TeV}, v_\eta = 2 \text{ TeV},$$

for phenomenological reasons, which we will discuss in the following sections.

2.1. Dark matter mass

In our model we treat the neutral components $L_{4L,4R}^0$ of L_4 as the dark matter. There are both Dirac as well as Majorana mass terms possible for (L_{4L}^0, L_{4R}^0) and given by the Lagrangian,

$$- \mathcal{L}_{L_4} = y_{\chi_D} \bar{L}_{4L} L_{4R} \eta + y_{\chi_D} \bar{L}_{4R} L_{4L} \eta + \frac{y_{\chi_L}}{\sqrt{2}} \bar{L}_{4L}^c i\tau_2 \Delta L_{4L} + \frac{y_{\chi_R}}{\sqrt{2}} \bar{L}_{4R}^c i\tau_2 \Delta L_{4R} \quad (12)$$

where y_{χ_D} and $y_{\chi_{(L,R)}}$ are the Yukawa couplings of vector-like leptons with η and Δ respectively. After the scalars η and Δ take their vevs, they generate Dirac mass $m_{\chi_D} = y_{\chi_D} v_\eta$ and Majorana mass $M_{\chi_{M(L,R)}} = y_{\chi_{(L,R)}} v_\Delta$. The Dirac mass term similar to Eq.(12) also generate the mass for charged lepton L_4^- . In the (L_{4L}^0, L_{4R}^0) basis, the mass matrix of L_4^0 is,

$$M_\chi = \begin{pmatrix} M_{\chi_{ML}} & m_{\chi_D} \\ m_{\chi_D} & M_{\chi_{MR}} \end{pmatrix},$$

Since the triplet vev $v_\Delta \ll v_\eta$, we assume that $m_{\chi_D} \gg M_{\chi_{ML}}, M_{\chi_{MR}}$. The mass eigenstates are pseudo-Dirac with eigenvalues,

$$m_{\chi_1} = m_{\chi_D} - \delta_\chi \quad (13)$$

$$m_{\chi_2} = m_{\chi_D} + \delta_\chi \quad (14)$$

with $\delta_\chi = (M_{\chi_{ML}} + M_{\chi_{MR}})/2$, where δ_χ denotes the mass splitting between the states χ_1 and χ_2 . The corresponding mass eigenstates are,

$$\chi_1 \simeq \frac{i(-L_{4L}^0 + L_{4R}^0)}{\sqrt{2}} + \frac{i(M_{\chi_{ML}} - M_{\chi_{MR}})}{2m_{\chi_D}} L_{4L}^0 \quad (15)$$

$$\chi_2 \simeq \frac{(L_{4L}^0 + L_{4R}^0)}{\sqrt{2}} + \frac{(M_{\chi_{ML}} - M_{\chi_{MR}})}{2m_{\chi_D}} L_{4L}^0 \quad (16)$$

The coupling of vector doublets to the Z boson is given as,

$$\bar{L}_4^0 \gamma^\mu L_4^0 Z_\mu = i(\bar{\chi}_1 \bar{\sigma}^\mu \chi_2 - \bar{\chi}_2 \bar{\sigma}^\mu \chi_1) Z_\mu + \frac{\delta_\chi}{2m_{\chi_D}} (\bar{\chi}_2 \bar{\sigma}^\mu \chi_2 - \bar{\chi}_1 \bar{\sigma}^\mu \chi_1) Z_\mu \quad (17)$$

We assume that $\delta_\chi \sim 100$ KeV so that the Z mediated scattering between the dark matter χ_1 and nuclei is suppressed to evade the bounds from direct detection experiments [28, 29].

2.2. Neutrino mass

In our model we introduced right-handed neutrinos that can mix with the neutral component of the vector-like lepton. In the presence of ν_{iR} ($i = e, \mu, \tau$), neutrinos will contain a Dirac mass in addition to the Majorana mass and their masses will generate by Type-II see-saw mechanism. The Lagrangian that gives mass to the neutrinos is given by,

$$\mathcal{L}_\nu = y_{\nu_D}^{ij} \bar{L}_L^i \tilde{\phi} \nu_R^j + y_{\nu_R}^{ij} \bar{\nu}_R^{ic} \nu_R^j \eta + \frac{y_{\nu_L}^{ij}}{\sqrt{2}} \bar{L}_L^{ic} i\tau_2 \Delta L_L^j + \frac{y_{\nu_4}}{\sqrt{2}} \bar{L}_L^{ic} i\tau_2 \Delta L_{4L} + h.c. \quad (18)$$

where y_{ν_D} , y_{ν_4} and $y_{\nu_{(L,R)}}$ are the Yukawa couplings of leptons to the scalars ϕ , η and Δ respectively. After the scalars take their vevs, they generate Dirac mass $m_{\nu_D} = y_{\nu_D} v_\phi$ and Majorana mass $M_{\nu_{(L,R)}} = y_{\nu_{(L,R)}} v_\eta$. In the basis (ν_L, ν_R) , the 6×6 mass matrix of ν is,

$$M_\nu = \begin{pmatrix} M_{\nu_L} & m_{\nu_D}^T \\ m_{\nu_D} & M_{\nu_R} \end{pmatrix}, \quad (19)$$

where m_{ν_D} and M_{ν_R} are the 3×3 Dirac and Majorana matrices respectively. After diagonalizing the mass matrix M_ν , the light neutrino mass is given by [30],

$$m_\nu = M_{\nu_L} - m_{\nu_D}^T M_{\nu_R}^{-1} m_{\nu_D} + \frac{1}{2} m_{\nu_D}^T M_{\nu_R}^{-1} X M_{\nu_R}^{-1} m_{\nu_D} - \frac{1}{2} (C + C^T), \quad (20)$$

where $C = m_{\nu_D}^T M_{\nu_R}^{-1} (M_{\nu_R}^*)^{-1} m_{\nu_D}^* M_{\nu_L}$ and $X = A + A^T$ with $A \equiv m_{\nu_D} m_{\nu_D}^\dagger (M_{\nu_R}^*)^{-1}$. We choose a m_{ν_D} texture such that the leading term $m_{\nu_D}^T M_{\nu_R}^{-1} m_{\nu_D} = 0$ and the neutrino mass is given by the second order term in $M_{\nu_R}^{-1}$. In this way one can get realistic neutrino mass from TeV scale vev of η .

g_H	m_{χ_1}	δ_χ	$m_{L_4^-}$	M_{V^+, V_3}
0.090	1000 GeV	100 KeV	1100 GeV	2000 GeV

TABLE III: Numerical values of the parameters used in our model.

3. DARK MATTER PHENOMENOLOGY

In our model, we identify the 4th generation neutral fermions ($L_{4L,4R}^0$) as the dark matter, and take their masses ~ 1 TeV, which is required to fit AMS-02 data [15]. The dominant channels of dark matter annihilation into the standard model particles are shown in Fig.(1). In this scenario for getting the correct relic density, we consider the resonance channel by taking $M_{V_3} \approx 2m_{\chi_1}$. The annihilation CS for relic density comes out $O \sim 10^{-26} \text{cm}^3 \text{s}^{-1}$ with the resonant enhancement. We assume the galactic rms velocity $v \sim 10^{-3}$ and get a further enhancement by the Sommerfeld mechanism [27] to get a CS of $O \sim 10^{-24} \text{cm}^3 \text{s}^{-1}$ which contributes to AMS-02 [15]. In the following sections we will discuss this scenario in details.

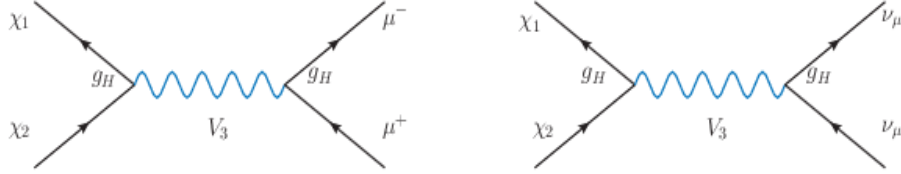


FIG. 1: Feynman diagrams of dark matter annihilation.

3.1. Relic density

The principal annihilation channels for our model are shown in Fig.(1). The combined annihilation cross-section for these channels is,

$$\sigma v = \frac{g_H^4}{8\pi} \frac{m_{\chi_1}^2}{(s - M_{V_3}^2)^2 + \Gamma_{V_3}^2 M_{V_3}^2} \quad (21)$$

where g_H is the horizontal gauge boson coupling to the dark matter, m_{χ_1} the dark matter mass, M_{V_3} and Γ_{V_3} are the mass and the decay width of new gauge boson respectively. The main contributions to the decay width of V_3 come from the decay modes $V_3 \rightarrow \mu^+ \mu^-$, $\nu_\mu \bar{\nu}_\mu$ and given as,

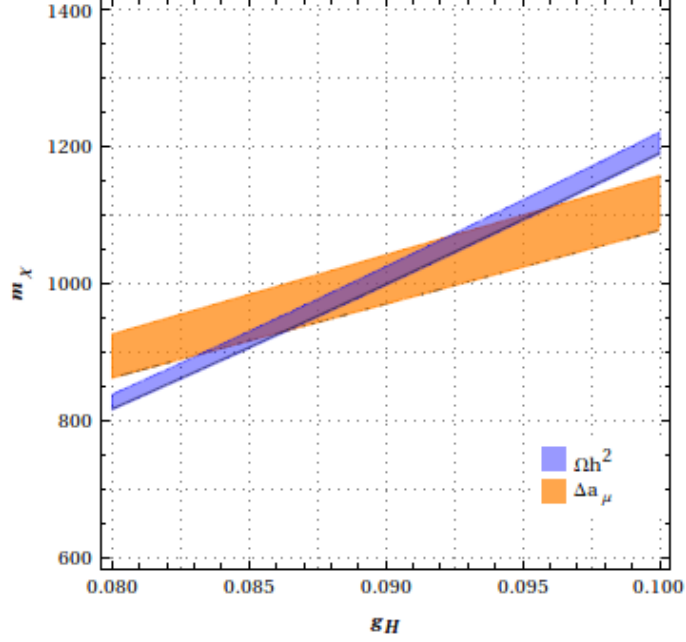


FIG. 2: Relic density with muon MDM within 1σ for $m_{L_4^-} = 1100$ GeV. The overlapping region is the part of parameter space, in which relic density and muon MDM constraints satisfied simultaneously.

$$\Gamma_{V_3} = \frac{g_H^2}{48\pi} M_{V_3} \quad (22)$$

In the non-relativistic limit, $s = 4m_{\chi_1}^2 + m_{\chi_1}^2 v^2 + 4m_{\chi_1} \delta_\chi$, where δ_χ is the mass splitting between the dark matter mass eigenstates (Eq.13-14). The thermal average of annihilation rate is given as [21–23],

$$\langle \sigma v \rangle = \frac{1}{n_{EQ}^2} \frac{m_{\chi_1}}{64\pi^4 x} \int_{4m_{\chi_1}^2}^{\infty} \hat{\sigma}(s) \sqrt{s} K_1 \left(\frac{x\sqrt{s}}{m_{\chi_1}} \right) ds, \quad (23)$$

where,

$$n_{EQ}^2 = \frac{g_i}{2\pi^2} \frac{m_{\chi_1}^3}{x} K_2(x), \quad (24)$$

$$\hat{\sigma}(s) = 2g_i^2 m_{\chi_1} \sqrt{s - 4m_{\chi_1}^2} \sigma v, \quad (25)$$

where $x = m_{\chi_1}/T$; $K_1(x)$, $K_2(x)$ represent the modified Bessel functions of the second type and g_i is the internal degree of freedom of DM particle. Using Eq.(24) and Eq.(25) in Eq.(23), it can be written as,

$$\langle \sigma v \rangle = \frac{g_H^4 m_{\chi_1}^2}{32} \frac{x^{3/2}}{\pi^{3/2}} \int_0^{\infty} \frac{\sqrt{z} \text{Exp}[-xz/4]}{(4m_{\chi_1}^2 + m_{\chi_1}^2 z + 4m_{\chi_1} \delta_\chi - M_{V_3}^2)^2 + \Gamma_{V_3}^2 M_{V_3}^2} dz \quad (26)$$

where $z = v^2$. We solve the integral for freeze-out ($x = 20$) condition with the parameters given in Table(III) and found $\langle\sigma v\rangle \sim O(10^{-26}\text{cm}^3\text{s}^{-1})$ which gives the correct relic density $\Omega h^2 = 0.1199 \pm 0.0027$, consistent with Planck [25] and WMAP [26] data as shown in Fig.(2). To satisfy AMS-02 data [15], we need a boost factor of $\sim O(100)$ [18–20]. We use the Sommerfeld enhancement [27] for getting the needed boost factor that we will describe in the following section.

3.2. Sommerfeld Enhancement

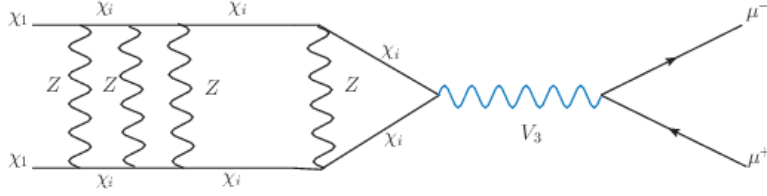


FIG. 3: Feynman diagram for the process $\chi_1\chi_1 \rightarrow \mu^+\mu^-$, which gives rise to Sommerfeld enhancement.

In a non-relativistic annihilation process there can be a distribution of the initial states wave functions by the exchange of light particles. In the case of $\chi_1\chi_1 \rightarrow \mu^+\mu^-$ annihilation, the Z -exchange ladder (Fig.3) gives a Sommerfeld enhancement [27] to the CS. The Schrodinger eq. obeyed by the dark matter doublet $\psi = (\chi_1, \chi_2)$ is

$$-\frac{1}{2M}\nabla^2\psi + V(r)\psi = \frac{k^2}{2M}\psi \quad (27)$$

where the 2×2 matrix potential in the (χ_1, χ_2) basis is,

$$V = \begin{pmatrix} 0 & -\frac{\alpha_z}{r}e^{-M_z r} \\ -\frac{\alpha_z}{r}e^{-M_z r} & \delta_\chi \end{pmatrix}, \quad (28)$$

The potential energy term mixes the χ_1, χ_2 states. The $\chi_1\chi_1 \rightarrow \mu^+\mu^-$ annihilation CS is enhanced by the Sommerfeld factor S ,

$$\sigma(\chi_1\chi_1 \rightarrow \mu^+\mu^-) = S \sigma_0(\chi_1\chi_1 \rightarrow \mu^+\mu^-) \quad (29)$$

where σ_0 is the CS without the Z exchange ladder and S is determined from the solution of the Schrodinger equation and defined as,

$$S = \left| \frac{\psi(\infty)}{\psi(0)} \right|^2 \quad (30)$$

The S factor of the inelastic Sommerfeld enhancement process has been computed in [27]. The Sommerfeld enhancement depends on the relativistic velocity $v \sim (10^{-3}-10^{-4})$, Z coupling α_z , Z mass M_Z , the dark matter mass m_{χ_1} and mass splitting δ_χ . For the parameters of our model (Table.III), the Sommerfeld enhancement factor is given by [27],

$$S = \frac{2\pi}{\epsilon_v} \sinh\left(\frac{\epsilon_v \pi}{\mu}\right) \frac{\cosh\left(\left(\epsilon_v + \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right) \pi/2\mu\right) \operatorname{sech}\left(\left(\epsilon_v - \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right) \pi/2\mu\right)}{\cosh\left(\left(\epsilon_v + \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right) \pi/\mu\right) - \cos(2\theta_-)} \quad (31)$$

where away from the resonance $\theta_- = n\pi + \pi\sqrt{\epsilon_\delta^2 - \epsilon_v^2}/2\mu$. The dimensionless parameters are defined as $\epsilon_v = v/\alpha_z$, $\epsilon_\delta = \sqrt{\delta_\chi/m_{\chi_1}}/\alpha_z$ and $\mu \sim \epsilon_z = (M_Z/m_{\chi_1})/\alpha_z$. By using the values of parameters from Table(III), we get $\epsilon_v = 0.16$, $\epsilon_\delta = 0.007$ and $\epsilon_z = 2$. We use these values of dimensionless parameter in Eq.(31) and find the Sommerfeld enhancement factor $S = 105$, which is used as an input for computing the positron flux for comparison with the AMS-02 data [15].

3.3. Comparison with AMS-02 and PAMELA data

We use annihilation CS of $\chi_1\chi_2 \rightarrow \mu^+\mu^-$ mediated by V_3 at resonance when $M_{V_3} \approx 2m_{\chi_1}$, to get the correct relic density [25, 26], and Sommerfeld enhancement [27] for getting enhanced CS required for AMS-02 [15]. We use publicly available code PPPC4DMID [33, 34] to compute the positron spectrum $\frac{dN_{e^+}}{dE}$ from the decay of $\mu^+\mu^-$ for 1 TeV dark matter. We then use the GALPROP code [35, 36] in which we take the annihilation CS ($\chi_1\chi_1 \rightarrow \mu^+\mu^-$) and the positron spectrum $\frac{dN_{e^+}}{dE}$ as an input to the source term,

$$Q_{e^+}(E, \vec{r}) = \frac{\rho^2}{m_\chi^2} \sum_f \langle \sigma v \rangle_f \frac{dN_{e^+}^f}{dE} \quad (32)$$

where $\langle \sigma v \rangle_f$ denotes the annihilation-rate, $\frac{dN_{e^+}^f}{dE}$ positron spectrum and ρ denotes the density of dark matter in the Milky Way halo, which is described by NFW profile [37] in our model,

$$\rho_{\text{NFW}} = \rho_0 \frac{r_s}{r} \left(1 + \frac{r}{r_s}\right)^{-2}, \quad \rho_0 = 0.4 \text{ GeV/cm}^3, \quad r_s = 20 \text{ kpc}, \quad (33)$$

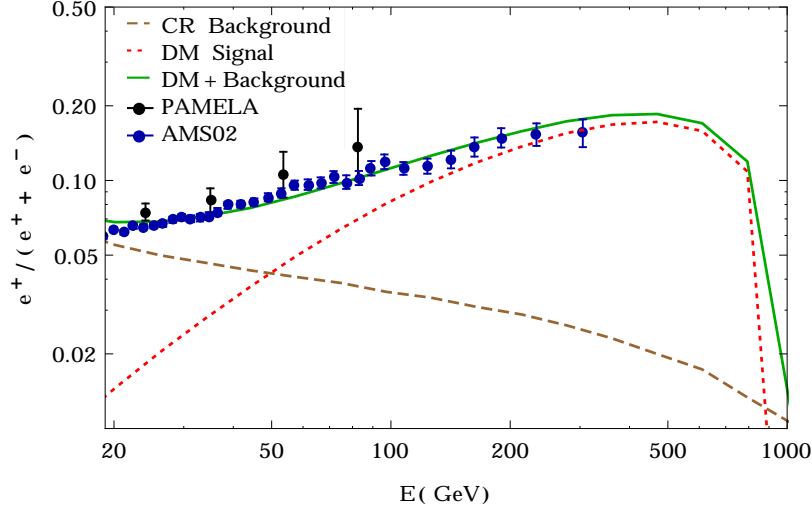


FIG. 4: The positron flux spectrum compared with data from AMS-02 [15] and PAMELA [16].

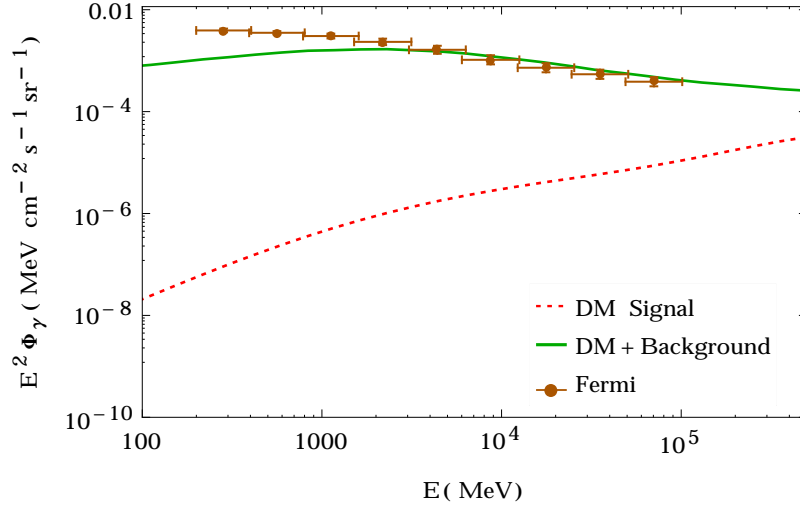


FIG. 5: The γ -ray spectrum compared with data from Fermi Lat [39].

The basic parameters of the GALPROP are chosen as in [31, 32] for 2D conventional model. In our model, we consider diffusion coefficient $D_0 = 2.8 \times 10^{28} \text{cm}^2 \text{s}^{-1}$ and Alfven speed $v_A = 30 \text{Kms}^{-1}$. We choose, $z_h = 4 \text{kpc}$ and $r_{max} = 20 \text{kpc}$, which are the half-width and maximum size for 2D galactic model respectively. In our model, the nucleus spectral index breaks at 9 GeV and spectral index above this is $\gamma_2^n = 2.36$ and below $\gamma_1^n = 1.82$. The normalization flux of electron at 100GeV is $1.25 \times 10^{-8} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1}$ and for the case

of electron, we take breaking point at 4 GeV and its injection spectral index above 4 GeV is $\gamma_1^{el} = 2.4$ and below $\gamma_0^{el} = 1.6$. After solving the propagation equation by taking the source term (Eq.32) and considering the Sommerfeld enhancement [27] of CS, GALPROP [35, 36] gives the desired positron flux. In Fig.(4), we plotted the output of GALPROP code and compared it with the observed AMS-02 data [15]. We see that our positron spectrum fits the AMS-02 data [15] very well. We also check, as shown in Fig.(5) that the γ - ray does not exceed the limits from Fermi-LAT [39] experiment. There is no antiproton excess in cosmic rays secondaries observed by PAMELA [38] and AMS-02 [24]. This fits in with the leptophilic DM of our model where only $\mu^+\mu^-$ is produced with an enhanced CS.

4. MUON MAGNETIC MOMENT

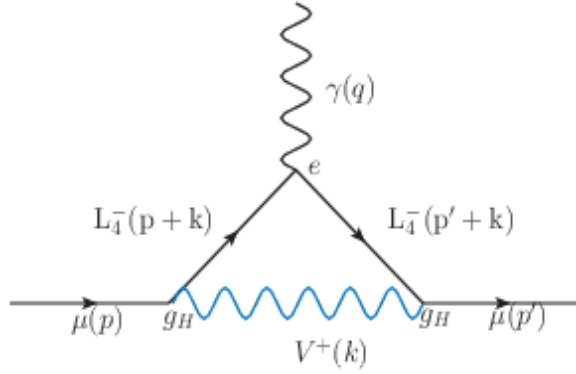


FIG. 6: Feynman diagram which gives contribution to muon $g - 2$, mediated by V^+ .

We calculate the magnetic moment operator, which is given by

$$\mathcal{L}_{MDM} = -ie\bar{\mu}(p')\frac{i}{2m_\mu}\sigma^{\mu\nu}F_2(q^2)\mu(p)F_{\mu\nu} \quad (34)$$

where $F_2(q^2)$ is the magnetic form factor, which arises from the loop. The anomalous magnetic moment is related to F_2 as $\Delta a_\mu = F_2(q^2 = 0)$. The vertex factor of the amplitude $\mu(p')\Gamma_\mu\mu(p)\epsilon^\mu$ shown in the Fig.(6) is,

$$\Gamma_\mu = \frac{eg_H^2}{2} \int \frac{d^4k}{(2\pi)^4} \gamma^\beta \frac{(\not{p}' + \not{k} + m_{L_4^-})}{(p' + k)^2 - m_{L_4^-}^2} \gamma_\mu \frac{(\not{p} + \not{k} + m_{L_4^-})}{(p + k)^2 - m_{L_4^-}^2} \gamma^\alpha \frac{g_{\alpha\beta}}{k^2 - M_{V^+}^2} \quad (35)$$

We perform the integration (see the Appendix) and use the Gordon identity to replace

$$(p_\mu + p'_\mu) = 2m_\mu\gamma_\mu + i\sigma^{\mu\nu}q_\nu. \quad (36)$$

and identify the coefficient of the $i\sigma^{\mu\nu}q_\nu$ as the magnetic form factor. In the limit of $M_{V^+}^2 \gg m_{L_4^-}^2$, we get the anomalous magnetic moment (see the Appendix).

$$\Delta a_\mu = F_2 = \frac{g_H^2}{8\pi^2} \left(\frac{m_\mu m_{L_4^-} - 2/3 m_\mu^2}{M_{V^+}^2} \right) \quad (37)$$

We note that in Eq.(37) the first term is dominant which shows that in the horizontal symmetry case there is a $m_\mu m_{L_4^-}$ enhancement over the model discussed in the introduction. We used the parameters from Table.(III) and found the muon MDM, in agreement with the experiment [1]. We plot the relic density and muon $g - 2$ together within 1σ in Fig.(2). This is clear from the plot that there exists a narrow region in which both constraints are satisfied together.

5. CONCLUSION

We have related two experimental signatures of possible beyond standard model physics in an canonical model involving an extra vector lepton doublet and a $SU(2_H)$ horizontal symmetry between the muon family and the L_4 family. An extra vector lepton doublet as asymmetric dark matter has been studied in [40, 41]. In our work we introduced horizontal gauge symmetry to get a contribution to the muon $g - 2$ and to explain AMS-02 data. This model requires $SU(2)$ triplet and singlet higgs of TeV scales and these can be probed in forthcoming LHC searches [42]. The charged fermions and scalars can also contribute to the $h \rightarrow \gamma\gamma$ loop [43, 44] and this can be a potential test of this model.

Appendix: Muon magnetic moment calculation

In our model contribution to the muon magnetic dipole moment comes from Fig.(6). The vertex factor of the amplitude is given in Eq.(35), which can be solved using standard Feynman parametrization method as follows,

$$\frac{N_\mu(k)}{[(p' + k)^2 - m_{L_4^-}^2][(p + k)^2 - m_{L_4^-}^2][k^2 - M_{V^+}^2]} = 2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \delta(1 - x - y - z) \frac{N_\mu(k)}{D^3}, \quad (A1)$$

where N_μ is the numerator in Eq.(35), and denominator is given by

$$\begin{aligned} D &= x[k^2 - M_{V+}^2] + y[(p+k)^2 - m_{L_4^-}^2] + z[(p'+k)^2 - m_{L_4^-}^2] \\ &= k^2 + 2k(y p + z p') + y p^2 + z p'^2 - (1-x)m_{L_4^-}^2 - x M_{V+}^2 \end{aligned} \quad (\text{A2})$$

where we use the property $x + y + z = 1$ because of δ -function appearing in Eq.(A1). To solve it further, we introduce a new momentum variable defined as,

$$l = k + y p + z p' \quad (\text{A3})$$

In terms of new variable, we can write Eq.(A2) as,

$$D = l^2 - \Delta, \quad (\text{A4})$$

where

$$\Delta = x M_{V+}^2 - x(1-x)m_\mu^2 + (1-x)m_{L_4^-}^2 \quad (\text{A5})$$

Here we use $p^2 = p'^2 = m_\mu^2$, which refers to $q^2 = 2m_\mu^2 - 2p \cdot p'$. In the numerator of Eq.(35) which is,

$$N_\mu(k) = \gamma^\beta \left[\not{p}' + \not{k} + m_{L_4^-} \right] \gamma_\mu \left[\not{p} + \not{k} + m_{L_4^-} \right] \gamma_\beta \quad (\text{A6})$$

we have to replace k by $l - y p - z p'$ and by this replacement, numerator N_μ becomes,

$$\begin{aligned} N_\mu(l - y p - z p') &= \gamma^\beta [(1-y)(1-z)\not{p}'\gamma_\mu\not{p} - z(1-z)\not{p}'\gamma_\mu\not{p}' + m_{L_4^-}(1-z)\not{p}'\gamma_\mu \\ &\quad - y(1-y)\not{p}\gamma_\mu\not{p} + y z \not{p}\gamma_\mu\not{p}' - m_{L_4^-} y \not{p}\gamma_\mu + \not{l}\gamma_\mu\not{l} \\ &\quad + m_{L_4^-}(1-y)\gamma_\mu\not{p} - m_{L_4^-} z \gamma_\mu\not{p}' + m_{L_4^-}^2 \gamma_\mu] \gamma_\beta. \end{aligned} \quad (\text{A7})$$

now we use *contraction identities* to simplify the numerator and it takes the following form,

$$\begin{aligned} N_\mu &= [\gamma_\mu(-2l^2 - 2m_{L_4^-}^2 - 2m_\mu^2 x^2 + 8m_\mu m_{L_4^-} x + 2q^2(1-y)(1-z)) \\ &\quad + i\sigma_{\mu\nu} q^\nu 4(1-y-z)(m_{L_4^-} - m_\mu(1-z))] \end{aligned} \quad (\text{A8})$$

where we used $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, and Gordon identity

$$(p_\mu + p'_\mu) = 2m_\mu \gamma_\mu + i\sigma_{\mu\nu} q^\nu \quad (\text{A9})$$

to replace $(p_\mu + p'_\mu)$. This is a general form of numerator and so we collect only the coefficient of $i\sigma_{\mu\nu}$ from Eq.(A8), which gives contribution to the anomalous magnetic moment of muon.

We use this coefficient,

$$4(1-y-z)(m_{L_4^-} - m_\mu(1-z)) \quad (\text{A10})$$

with Eq.(A4) in Eq.(35) and perform the integrations on variables y and z. At the last we get the following vertex factor,

$$\Gamma_\mu = eg_H^2 \int \frac{d^4l}{(2\pi)^4} \int_0^1 dx \frac{4m_{L_4^-}x(1-x) - 2m_\mu x(1-x^2)}{(l^2 - \Delta)^3} i\sigma^{\mu\nu} q_\mu \quad (\text{A11})$$

where $\Delta = xM_{V^+}^2 - x(1-x)m_\mu^2 + (1-x)m_{L_4^-}^2$, as defined in Eq.(A5). We do the Wick rotation to find the further solution through momentum integration, using the following identity

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{[l^2 - \Delta]^m} = \frac{i(-1)^m}{(4\pi)^2} \frac{1}{(m-1)(m-2)} \frac{1}{\Delta^{m-2}} \quad (\text{A12})$$

and get the vertex factor

$$\Gamma_\mu = \frac{-ieg_H^2}{32\pi^2} \int_0^1 dx \frac{4m_{L_4^-}x(1-x) - 2m_\mu x(1-x^2)}{\Delta} i\sigma^{\mu\nu} q_\mu. \quad (\text{A13})$$

We perform the integration in the limit of $M_{V^+}^2 \gg m_{L_4^-}^2$ and find the suitable form of vertex factor as follows,

$$\Gamma_\mu = -ie \frac{g_H^2}{8\pi^2} \frac{i}{2m_\mu} \left(\frac{m_\mu m_{L_4^-} - 2/3m_\mu^2}{M_{V^+}^2} \right) \sigma_{\mu\nu} q^\nu \quad (\text{A14})$$

after comparing our result with Eq.(34), we get the magnetic form factor, which in the limit of $q^2 = 0$ gives the muon anomalous magnetic moment,

$$\Delta a_\mu = F_2 = \frac{g_H^2}{8\pi^2} \left(\frac{m_\mu m_{L_4^-} - 2/3m_\mu^2}{M_{V^+}^2} \right) \quad (\text{A15})$$

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